HEAT TRANSFER DURING SHEET ROLLING

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An analytical solution to the heat conduction problem with a moving boundary is derived for the determination of principal temperatures in a metal during sheet rolling.

We will consider the problem of determining the metal temperature during sheet rolling. We assume that, before rolling, the billet constitutes an infinitely large plate of thickness 2R.

As the initial condition in this problem we take the temperature of the metal after it has been removed from the reheating furnace. It will be assumed, furthermore, that the temperature distribution across a billet section is at this instant parabolic:

$$T(x,\ 0)=T_{0c}+\Delta T_0\left(\frac{x}{R}\right)^2. \tag{1}$$

During a break period the mean thermal flux through the metal surface to the surrounding medium is q_1. During compression periods the mean (with respect to time) thermal flux from the metal to the rolls is q_2. Such a model of heat transfer between metal and surrounding medium is idealized. Actually, during a break period the heat is transferred from the metal according to the Newton law and the Stefan–Boltzmann law, while during compression the heat is transferred by conduction through the metal–scale–roll system. Of course, neither during the transport nor during the compression does the thermal flux remain constant with time.

On the other hand, the magnitudes of the thermal fluxes q_1 and q_2 depend also on the consecutive number of a given pass. Considering that q_1 and q_2 vary only slightly during a rolling operation, however, such an idealization of the process is permissible.

During compression there is heat released in the bulk of metal as a result of plastic deformation. Generally, this heat is distributed over the metal volume nonuniformly corresponding to the distribution of shear rate [1]. In our case, however, we ignore this and assume that during a compression period there is a uniformly distributed heat source within the metal volume acting with a power of W_2 W/m^3. During break periods the power of the heat source is $W_1 = 0$.

After each pass the thickness of a bar is reduced by the amount of compression, i.e., the billet edge advances according to some power law. For simplicity, we assume that the bar edge moves linearly at a speed of s m/sec.

In order to determine the temperature distribution function for the volume of rolled metal, we use the differential equation of heat conduction:

$$\frac{\partial T}{\partial t}=a\frac{\partial^2 T}{\partial x^2}+\frac{W(t)}{c\rho},\ 0\leqslant x\leqslant R-st \tag{2}$$

and the boundary conditions

$$\left.\frac{\partial T}{\partial x}\right|_{x=0}=0, \tag{3}$$

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$$\left.\frac{\partial T}{\partial x}\right|_{x=R-st} = -\frac{q(t)}{\lambda}, \tag{4}$$

where

$$W(t) = W_2 f(t),$$
$$q(t) = q_1 + (q_2 - q_1) f(t).$$

Function f(t) has the following values:

for the (m + 1)-th break period

$$f(t) = \sum_{1}^{m} \eta\,[t-(m-1)t_0 - t_1] - \eta\,(t - mt_0);$$

for the (m + 1)-th compression

$$f(t) = \sum_{0}^{m} \eta\,(t - mt_0 - t_1) - \sum_{1}^{m} \eta(t - mt_0).$$

The unity function η (z) is defined as

$$\eta(z) = \begin{cases} 0 & \text{for} \quad z < 0, \\ 1 & \text{for} \quad z \geqslant 0. \end{cases}$$

The solution to our problem will be sought in the following form:

$$T(x,\ t) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!a^n} \cdot \frac{d^n}{dt^n} B(t) + \frac{W(t)}{c\rho} t. \tag{5}$$

The boundary condition (4) will be rewritten as

$$\sum_{n=1}^{\infty} \frac{(R-st)^{2n-1}}{(2n)!a^n} \cdot \frac{d^n}{dt^n} B(t) = -\frac{q(t)}{\lambda}. \tag{6}$$

Starting out with (1), we assign the initial conditions for the B(t) function as

$$B(0) = T_{0C};\ \left.\frac{d}{dt} B(t)\right|_{t=0} = 2a\,\Delta\, T_0;\ \left.\frac{d^n}{dt^n} B(t)\right|_{\substack{t=0\\ n>1}} = 0. \tag{7}$$

We now introduce the dimensionless quantities:

$$X = \frac{x}{R};\ \tau = \frac{at}{R^2};\ v = \frac{T\lambda}{q_1 R};\ \beta(\tau) = \frac{B(\tau)\lambda}{q_1 R};$$
$$K(\tau) = \frac{W(\tau)R}{q_1};\ b = M\tau;\ M = \frac{sR}{a};\ Q(\tau) = \frac{q(\tau)}{q_1} \tag{7'}$$

to express (5) and (6) in dimensionless form:

$$v(X,\ \tau) = \sum_{n=0}^{\infty} \frac{X^{2n}}{(2n)!} \cdot \frac{d^n}{d\tau^n} \beta(\tau) + K(\tau)\,\tau, \tag{8}$$

$$\sum_{n=1}^{\infty} \frac{(1-b)^{2n-1}}{(2n-1)!} \cdot \frac{d^n}{d\tau^n} \beta(\tau) = -Q(\tau). \tag{9}$$

We introduce a tentative parameter ξ which characterizes the convergence and we rewrite (9) as

$$\sum_{n=1}^{\infty} \frac{(1-\xi b)^{2n-1}}{(2n-1)!} \cdot \frac{d^n}{d\tau^n} \beta(\tau) = -Q(\tau). \tag{10}$$

Function $\beta(\tau)$ will be approximated by the series

$$\beta(\tau) = \beta_0 + \xi\beta_1 + \xi^2\beta_2 + \ldots \tag{11}$$

Inserting (11) into (10) and comparing the coefficients of like power terms in ξ, we find successive approximations to the differential equation (9). For example, the zeroth approximation is

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)!} \cdot \frac{d^n}{d\tau^n} \beta_0(\tau) = -Q(\tau). \tag{12}$$

We will then solve the problem in the zeroth approximation. The initial conditions for function $\beta_0(\tau)$ will be

$$\beta_0(0) = v_{0C};\ \beta_0'(\tau)\Big|_{\tau=0} = 2\Delta v_0;\ \beta_0^{(n)}(\tau)\Big|_{\substack{\tau=0\\ n>1}} = 0. \tag{13}$$

The integral Laplace–Carson transformation is now applied to the differential equation (12). In the image plane we find the solution for function $\beta_0(\tau)$:

$$\bar{\beta}_0(p) = u_{0C} + \frac{2\Delta v_0}{p} - \frac{2\Delta v_0 + \bar{Q}(p)}{\sqrt{p}\,\mathrm{sh}\sqrt{p}}, \tag{14}$$

where

$$\bar{Q}(p) = 1 + (c' - 1)\bar{f}(p).$$

Function $\bar{f}(p)$ has the values:

for the (m + 1)-th break period

$$\bar{f}(p) = \sum_0^m \exp\{-p[(m-1)\tau_0 + \tau_1]\} - \exp(-pm\tau_0);$$

for the (m + 1)-th compression

$$\bar{f}(p) = \sum_0^m \exp[-p(m\tau_0 + \tau_1)] - \sum_1^m \exp(-pm\tau_0),$$

where p is a complex parameter.

With the aid of the lag theorem it is possible to obtain a solution in the region of the real variable.

For the first break period:

$$\beta_0(\tau) = \psi(\tau), \tag{15}$$

where

$$\psi(\tau) = v_{0C} + 2\Delta v_0 \tau - 2\Delta v_0 \Phi(\tau) - \Phi(\tau), \tag{16}$$

$$\Phi(\tau) = \tau - \frac{1}{6} - \frac{2}{\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^k \exp(-\pi^2 k^2 \tau)}{k^2}. \tag{17}$$

For the fist compression:

$$\beta_0(\tau) = \psi(\tau) - (c' - 1)\Phi(\tau - \tau_1)\eta(\tau - \tau_1). \tag{18}$$

By analogy, for the (m + 1)-th break period:

$$\beta_0(\tau) = \psi(\tau) - (c' - 1)\Big\{\sum_1^m \Phi[\tau - (m-1)\tau_0 - \tau_1]\eta[\tau - (m-1)\tau_0 - \tau_1] - \Phi(\tau - m\tau_0)\eta(\tau - m\tau_0)\Big\}, \tag{19}$$

and for the (m + 1)-th compression:

$$\beta_0(\tau) = \psi(\tau) - (c' - 1)\Big[\sum_0^m \Phi(\tau - m\tau_0 - \tau_1)\eta(\tau - m\tau_0 - \tau_1) - \sum_1^m \Phi(\tau - m\tau_0)\eta(\tau - m\tau_0)\Big]. \tag{20}$$

The rolling process is usually completed within short periods of time. Considering that the series in $\Phi(\tau)$ is not convergent enough at small values of τ, it is worthwhile to represent this function in a form convenient for calculations at such small time values. In accordance with [2], we perform the following transformation:

$$\frac{1}{\sqrt{p}\,\mathrm{sh}\sqrt{p}} = \frac{2\exp(-\sqrt{p})}{\sqrt{p}[1-\exp(-2\sqrt{p})]}.$$

Taking into account that $\exp(-2\sqrt{p}) < 1$, one can write

$$\frac{2\exp(-\sqrt{p})}{\sqrt{p}[1-\exp(-2\sqrt{p})]} = \frac{2\exp(-\sqrt{p})}{\sqrt{p}} \sum_{k=0}^{\infty} \exp(-2k\sqrt{p}). \tag{21}$$

Disregarding all terms after the first one in series (21), we obtain after inverse transformation:

$$\Phi(\tau) \approx 2\left[2\sqrt{\frac{\tau}{\pi}}\exp\left(-\frac{1}{4\tau}\right) - \mathrm{erfc}\left(\frac{1}{2\sqrt{\tau}}\right)\right]. \tag{22}$$

This expression is much more convenient than (17) for calculations at small time values.

Similarly one can determine the next approximations of function $\beta(\tau)$. Calculations have shown that this is not necessary, however, because the first approximation is almost the same as the zeroth one and, therefore, it is permissible to limit the calculations to the zeroth approximation.

An analysis has shown that the series in expression (8) is not convergent enough at values of X close to unity. At X = 0.9, for example, it was necessary to include 16 terms of the series before an accurate result could be obtained. It would be desirable to find a solution not as cumbersome as that. Considering that large values of X correspond to the first stage of the rolling process, such a solution can be found on the basis of the following conceptions.

In accordance with the engineering model of heat conduction in [3], a heat pulse transmitted to the billet surface reaches to center of the billet after an inertial time interval. In our case the body is a plate whose thickness is 2(R − st) and in which the temperature field is symmetrical. Within the inertial time interval it is almost immaterial whether in an analytical description of the temperature field in a layer R' = R − st thick this layer is assumed infinite or semiinfinite. Therefore, we will look for the temperature distribution function inside a semiinfinite body whose edge advances linearly. Within the inertial time interval this function will satisfactorily describe the temperature field in the bar at large values of X.

The differential equation of heat conduction

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial y} + \frac{W(t)}{c\rho}\ (st < y < \infty) \tag{23}$$

is solved with the constraints:

$$\lambda \frac{\partial T}{\partial y}\bigg|_{y=st} = q(t), \tag{24}$$

$$T(\infty, t) \neq \infty, \tag{25}$$

$$T(y, 0) = T_{0c} + \frac{(R-y)^2}{R^2}\Delta T_0. \tag{26}$$

The initial condition (26) is incompatible with condition (25), but of interest here is a metal layer with an initial thickness R and, therefore, we will disregard this contradiction.

In order to transform a moving temperature field in a stationary system of coordinates into a stationary field in a moving system of coordinates, we let [4, 5]

$$x_1 = y - st. \tag{27}$$

Using the dimensionless quantities, we obtain a system of dimensionless equations in a moving system of coordinates:

$$\frac{\partial^2 v}{\partial X_1^2} + M\frac{\partial v}{\partial X_1} + K(\tau) = \frac{\partial v}{\partial \tau}. \quad (0 \leqslant X_1 \leqslant \infty), \tag{28}$$

$$\left.\frac{\partial v}{\partial X_1}\right|_{X_1=0} = Q(\tau), \tag{29}$$

$$v(\infty, \tau) \neq \infty, \tag{30}$$

$$v(X_1, 0) = v_{0c} + \Delta v_0 (1 - X_1)^2, \tag{31}$$

where $X_1 = x_1/R$ and the other quantities are determined from (7'). (In this case R denotes the initial thickness of the metal layer.)

After performing the Laplace–Carson transformation on (28), we obtain the equation in operator form:

$$\frac{\partial^2 \bar{v}(X_1, p)}{\partial X_1^2} + M\frac{\partial \bar{v}(X_1 p)}{\partial X_1} - p\bar{v}(X_1, p) = -\bar{K}p - pv_{0c} - p\Delta v_0(1 - X_1)^2. \tag{32}$$

The solution to the problem in the image plane is written as

$$\bar{v}(X_1, p) - v_{0c} = \frac{2\Delta v_0 M \exp\left[-X_1\left(\frac{M}{2} + \sqrt{\frac{M^2}{4} + p}\right)\right]}{p\left(\frac{M}{2} + \sqrt{\frac{M^2}{4} + p}\right)}$$

$$- \frac{\bar{Q}(p) + 2\Delta v_0}{\frac{M}{2} + \sqrt{\frac{M^2}{4} + p}} \exp\left[-x_1\left(\frac{M}{2} + \sqrt{\frac{M^2}{4} + p}\right)\right] + \frac{\bar{K}(p)}{p}$$

$$+ \Delta v_0\left[\frac{2}{p}(1 - M + MX_1) + \frac{2M^2}{p^2} + (1 - X_1)^2\right]. \tag{33}$$

A subsequent inverse transformation yields for the (m + 1)-th break period

$$v(X_1, \tau) - v_{0c} = 2\Delta v_0 MN(\tau) - 2\Delta v_0 \psi(\tau) - \psi(\tau)$$

$$-(c' - 1)\sum_1^m \psi(z_1)\eta(z_1) - \psi(z_2)\eta(z_2) + K_2\sum_1^m z_1\eta(z_1) - z_2\eta(z_2)$$

$$+ \Delta v_0[2(1 - M + MX_1)\tau + M^2\tau^2 + (1 - X_1)^2], \tag{34}$$

and for the (m + 1)-th compression

$$v(X, \tau) - v_{0c} = 2\Delta v_0 MN(\tau) - 2\Delta v_0 \psi(\tau) - \psi(\tau)$$

$$-(c' - 1)\left[\sum_0^m \psi(z_3)\eta(z_3) - \sum_1^m \psi(z_2)\eta(z_2)\right] + K_2\left[\sum_0^m z_3\eta(z_3) - \sum_1^m z_2\eta(z_2)\right]$$

$$+ \Delta v_0[2(1 - M + MX_1)\tau + M^2\tau^2 + (1 - X_1)^2], \tag{35}$$

where

$$\psi(\tau) = \int_0^\tau \left[\frac{\exp\left[-\left(\frac{M}{2}X_1 + \frac{X_1^2}{4t} + \frac{M^2}{4}t\right)\right]}{\sqrt{\pi t}} - \frac{M}{2}\operatorname{erfc}\left(\frac{X_1}{2\sqrt{t}} + \frac{M}{2}\sqrt{t}\right)\right]dt;$$

$$N(\tau) = \int_0^\tau \psi(t)\,dt;$$

$$z_1 = \tau - (m - 1)\tau_0 - \tau_1; \quad z_2 = \tau - m\tau_0; \quad z_3 = \tau - m\tau_0 - \tau_1.$$

Expressions (34) and (35) are more convenient than function (8) for calculations with $X \to 1$ (which corresponds to $X_1 \to 0$).

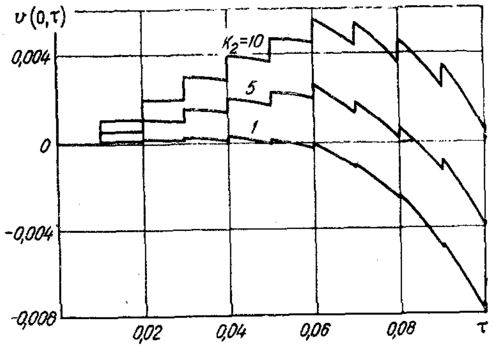

Fig. 1

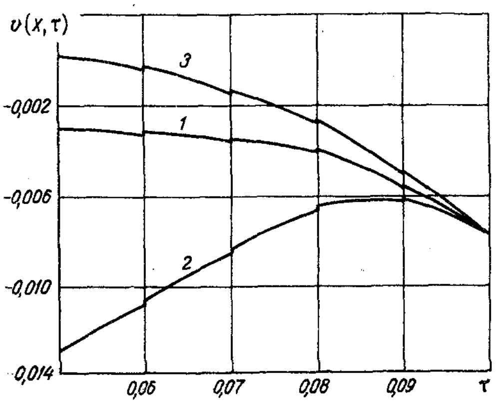

Fig. 2

Fig. 1. Temperature at the center of a bar as a function of time, at various values of the K_2 number.

Fig. 2. Mean temperature (1), surface temperature (2) and temperature at the center (3) of a bar, as functions of time for $K_2 = 1$.

Of special practical interest is a calculation of the mean (over the cross section) bar temperature during passes. An expression for this mean temperature can be obtained simply by integrating (8) over the interval from 0 to (1 – b):

$$v_{av} = \frac{1}{1-b}\int_0^{1-b}\sum_{n=0}^{\infty}\left[\frac{X^{2n}}{(2n)!}\cdot\frac{d^n}{d\tau^n}\beta(\tau) + K(\tau)\tau\right]dX$$

$$= \sum_{n=0}^{\infty}\frac{(1-b)^{2n}}{(2n+1)!}\cdot\frac{d^n}{d\tau^n}\beta(\tau) + K(\tau)\tau. \qquad (36)$$

The bar temperature in dimensionless values was calculated for individual passes using Eqs. (8), (9), (20), (34), (35), and (36). The values of the dimensionless quantities characterizing the rolling process were assumed: $\Delta v_0 = 0$, $M = 10$, $\tau_1 = 0.0099$, $\tau_2 = 0.0001$, number of passes 10, $c' = 20$, metal thickness after rolling $X = 0.083$.

The results of calculations are shown in Figs. 1 and 2. The following can be concluded from Fig. 1. During each pass the temperature at the center of a bar rises owing to the heat given off by the metal as a result of plastic deformation. After the inertial time interval has elapsed ($\tau \approx 0.04$), the center layer of the bar begins to cool down as a result of a heat transfer from the metal surface to the surrounding medium. Depending on the consecutive number of a given pass and also on the ratio of heat generated during compression to heat lost to the surrounding medium, the temperature at the center of a billet will be either higher or lower than initially.

As the rolling process nears completion, the temperatures at all points of a bar tend toward the mean temperature (Fig. 2.).

While the principal temperatures of a rolled metal are by the conventional method calculated from the mean-mass-temperature, the method proposed here makes it possible to account for the nonuniform temperature distribution across the bar height.

NOTATION

$T(x, t)$	metal temperature;
x	linear coordinate;
t	time;
T_{0c}	temperature at center of billet when removed from furnace;
ΔT_0	initial temperature drop across billet cross section;
a	thermal diffusivity;

c	specific heat of metal;
ρ	density of metal;
λ	thermal conductivity of metal;
t_1	average duration of break period;
t_2	average time of contact between metal and roll;

$t_0 = t_1 + t_2$;
$c' = q_2/q_1$.

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